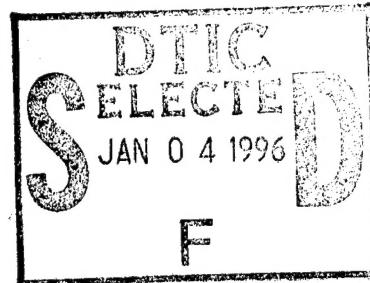


NAVAL POSTGRADUATE SCHOOL

Monterey, California



**SOFTWARE FOR THE STAGGERED AND
UNSTAGGERED TURKEL-ZWAS SCHEMES FOR THE
SHALLOW WATER EQUATIONS ON THE SPHERE**

by

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13. ABSTRACT (Maximum 200 words) <p>A linear analysis of the shallow water equations in spherical coordinates for the Turkel-Zwas¹ explicit large time-step scheme was presented by Neta, Giraldo and Navon² as well as the unstaggered¹ Turkel-Zwas scheme for the solution of the shallow water equations on the sphere.</p>				
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1. INTRODUCTION

In this paper we present the software developed for the solution of the shallow water equations in spherical coordinates. The unstaggered (original) Turkel-Zwas scheme¹ and the staggered² one are both given.

The shallow water equations in spherical coordinates are given by

$$\frac{\partial u}{\partial t} + \frac{1}{a \cos \theta} \left[u \frac{\partial u}{\partial \lambda} + v \cos \theta \frac{\partial u}{\partial \theta} \right] - \left(f + \frac{u}{a} \tan \theta \right) v + \frac{g}{a \cos \theta} \frac{\partial h}{\partial \lambda} = 0 \quad (1)$$

$$\frac{\partial v}{\partial t} + \frac{1}{a \cos \theta} \left[u \frac{\partial v}{\partial \lambda} + v \cos \theta \frac{\partial v}{\partial \theta} \right] + \left(f + \frac{u}{a} \tan \theta \right) u + \frac{g}{a} \frac{\partial h}{\partial \theta} = 0 \quad (2)$$

$$\frac{\partial h}{\partial t} + \frac{1}{a \cos \theta} \left[\frac{\partial}{\partial \lambda} (hu) + \frac{\partial}{\partial \theta} (hv \cos \theta) \right] = 0. \quad (3)$$

Here, f is the Coriolis parameter given by

$$f = 2\Omega \sin \theta \quad (4)$$

where Ω is the angular speed of the rotation of the earth, h is the height of the homogeneous atmosphere, u and v are the zonal and meridional wind components respectively, θ and λ are the latitudinal and longitudinal directions respectively, a is the radius of the earth, and g is the gravitational constant.

In section 2 we present the unstaggered scheme (modified as suggested by Neta³). In section 3 we present the staggered method as developed by Neta, Giraldo and Navon². In section 4 we present the input file required including a logical parameter to choose between the staggered and unstaggered versions. In section 5 we present the code developed.

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2. UNSTAGGERED TURKEL-ZWAS SCHEME

The Turkel-Zwas scheme for the nonlinear shallow water equations in spherical coordinates takes the following form:

$$\begin{aligned}
u_{k,j}^{\ell+1} = & u_{k,j}^{\ell-1} - \sigma \left[\frac{u_{k,j}^{\ell}}{\cos \theta_j} (u_{k+1,j}^{\ell} - u_{k-1,j}^{\ell}) + v_{k,j}^{\ell} (u_{k,j+1}^{\ell} - u_{k,j-1}^{\ell}) \right. \\
& \left. + \frac{g}{p \cos \theta_j} (h_{k+p,j}^{\ell} - h_{k-p,j}^{\ell}) \right] \\
& + 2\Delta t \left[(1 - \alpha) (f_j + \frac{u_{k,j}^{\ell}}{a} \tan \theta_j) v_{k,j}^{\ell} \right. \\
& \left. + \frac{\alpha}{2} (f_j + \frac{u_{k+p,j}^{\ell}}{a} \tan \theta_j) v_{k+p,j}^{\ell} \right. \\
& \left. + \frac{\alpha}{2} (f_j + \frac{u_{k-p,j}^{\ell}}{a} \tan \theta_j) v_{k-p,j}^{\ell} \right]
\end{aligned} \tag{5}$$

$$\begin{aligned}
v_{k,j}^{\ell+1} = & v_{k,j}^{\ell-1} - \sigma \left[\frac{u_{k,j}^{\ell}}{\cos \theta_j} (v_{k+1,j}^{\ell} - v_{k-1,j}^{\ell}) + v_{k,j}^{\ell} (v_{k,j+1}^{\ell} - v_{k,j-1}^{\ell}) \right. \\
& \left. + \frac{g}{q} (h_{k,j+q}^{\ell} - h_{k,j-q}^{\ell}) \right] \\
& - 2\Delta t \left[(1 - \alpha) \left(f_j + \frac{u_{k,j}^{\ell}}{a} \tan \theta_j \right) u_{k,j}^{\ell} \right. \\
& \left. + \frac{\alpha}{2} \left(f_{j+q} + \frac{u_{k,j+q}^{\ell}}{a} \tan \theta_{j+q} \right) u_{k,j+q}^{\ell} \right. \\
& \left. + \frac{\alpha}{2} \left(f_{j-q} + \frac{u_{k,j-q}^{\ell}}{a} \tan \theta_{j-q} \right) u_{k,j-q}^{\ell} \right]
\end{aligned} \tag{6}$$

$$\begin{aligned}
h_{k,j}^{\ell+1} = & h_{k,j}^{\ell-1} - \sigma \left\{ \frac{u_{k,j}^{\ell}}{\cos \theta_j} (h_{k+1,j}^{\ell} - h_{k-1,j}^{\ell}) + v_{k,j}^{\ell} (h_{k,j+1}^{\ell} - h_{k,j-1}^{\ell}) \right. \\
& + \frac{h_{k,j}^{\ell}}{\cos \theta_j} \left[(1 - \alpha) (u_{k+p,j}^{\ell} - u_{k-p,j}^{\ell}) \right. \\
& \left. + \frac{\alpha}{2} (u_{k+p,j+q}^{\ell} - u_{k-p,j+q}^{\ell} + u_{k+p,j-q}^{\ell} - u_{k-p,j-q}^{\ell}) \right] \frac{1}{p} \\
& + \frac{h_{k,j}^{\ell}}{\cos \theta_j} \left[(1 - \alpha) (v_{k,j+q}^{\ell} \cos \theta_{j+q} - v_{k,j-q}^{\ell} \cos \theta_{j-q}) \right. \\
& \left. + \frac{\alpha}{2} (v_{k+p,j+q}^{\ell} \cos \theta_{j+q} - v_{k+p,j-q}^{\ell} \cos \theta_{j-q} + v_{k-p,j+q}^{\ell} \cos \theta_{j+q} - v_{k-p,j-q}^{\ell} \cos \theta_{j-q}) \right] \frac{1}{q} \}
\end{aligned} \tag{7}$$

where

$$\sigma = \frac{\Delta t}{a\Delta\lambda} = \frac{\Delta t}{a\Delta\theta}. \quad (8)$$

For $\alpha = \frac{1}{3}$ the geostrophic balance and the incompressibility condition are satisfied to a higher order in the Cartesian coordinate case (See Turkel and Zwas,¹ Navon and de Villiers⁵).

Note that there is a typo in equation (11a) of Turkel-Zwas¹ which is our equation (5). We have also modified (to get a symmetric approximation as suggested by Neta⁴ for a rectangular domain) the right hand side of (11c) in Turkel-Zwas¹ which is (7) here.

3. STAGGERED TURKEL-ZWAS SCHEME

The staggered version of the Turkel-Zwas scheme as proposed by Neta, Giraldo and Navon² takes the following form:

$$\begin{aligned}
u_{k,j}^{\ell+1} = & u_{k,j}^{\ell-1} - \sigma \left[\frac{u_{k,j}^{\ell}}{\cos \theta_j} (u_{k+1,j}^{\ell} - u_{k-1,j}^{\ell}) + v_{k,j}^{\ell} (u_{k,j+1}^{\ell} - u_{k,j-1}^{\ell}) \right. \\
& \left. + \frac{2g}{p \cos \theta_j} (h_{k+\frac{p}{2},j}^{\ell} - h_{k-\frac{p}{2},j}^{\ell}) \right] \\
& + 2\Delta t \left[(1 - \alpha) (f_j + \frac{u_{k,j}^{\ell}}{a} \tan \theta_j) v_{k,j}^{\ell} \right. \\
& \left. + \frac{\alpha}{2} (f_j + \frac{u_{k+\frac{p}{2},j}^{\ell}}{a} \tan \theta_j) v_{k+\frac{p}{2},j}^{\ell} \right. \\
& \left. + \frac{\alpha}{2} (f_j + \frac{u_{k-\frac{p}{2},j}^{\ell}}{a} \tan \theta_j) v_{k-\frac{p}{2},j}^{\ell} \right]
\end{aligned} \tag{9}$$

$$\begin{aligned}
v_{k,j}^{\ell+1} = & v_{k,j}^{\ell-1} - \sigma \left[\frac{u_{k,j}^{\ell}}{\cos \theta_j} (v_{k+1,j}^{\ell} - v_{k-1,j}^{\ell}) + v_{k,j}^{\ell} (v_{k,j+1}^{\ell} - v_{k,j-1}^{\ell}) \right. \\
& \left. + \frac{2g}{q} (h_{k,j+\frac{q}{2}}^{\ell} - h_{k,j-\frac{q}{2}}^{\ell}) \right] \\
& - 2\Delta t \left[(1 - \alpha) \left(f_j + \frac{u_{k,j}^{\ell}}{a} \tan \theta_j \right) u_{k,j}^{\ell} \right. \\
& \left. + \frac{\alpha}{2} \left(f_{j+\frac{q}{2}} + \frac{u_{k,j+\frac{q}{2}}^{\ell}}{a} \tan \theta_{j+\frac{q}{2}} \right) u_{k,j+\frac{q}{2}}^{\ell} \right. \\
& \left. + \frac{\alpha}{2} \left(f_{j-\frac{q}{2}} + \frac{u_{k,j-\frac{q}{2}}^{\ell}}{a} \tan \theta_{j-\frac{q}{2}} \right) u_{k,j-\frac{q}{2}}^{\ell} \right]
\end{aligned} \tag{10}$$

$$\begin{aligned}
h_{k,j}^{\ell+1} = & h_{k,j}^{\ell-1} - \sigma \left\{ \frac{u_{k,j}^{\ell}}{\cos \theta_j} (h_{k+1,j}^{\ell} - h_{k-1,j}^{\ell}) + v_{k,j}^{\ell} (h_{k,j+1}^{\ell} - h_{k,j-1}^{\ell}) \right. \\
& + \frac{2h_{k,j}^{\ell}}{\cos \theta_j} \left[(1 - \alpha) (u_{k+\frac{p}{2},j}^{\ell} - u_{k-\frac{p}{2},j}^{\ell}) \right. \\
& \left. + \frac{\alpha}{2} (u_{k+\frac{p}{2},j+\frac{q}{2}}^{\ell} - u_{k-\frac{p}{2},j+\frac{q}{2}}^{\ell} + u_{k+\frac{p}{2},j-\frac{q}{2}}^{\ell} - u_{k-\frac{p}{2},j-\frac{q}{2}}^{\ell}) \right]^{\frac{1}{p}} \\
& \left. + \frac{2h_{k,j}^{\ell}}{\cos \theta_j} \left[(1 - \alpha) (v_{k,j+\frac{q}{2}}^{\ell} \cos \theta_{j+\frac{q}{2}} - v_{k,j-\frac{q}{2}}^{\ell} \cos \theta_{j-\frac{q}{2}}) \right. \right. \\
& \left. \left. + \frac{\alpha}{2} (v_{k+\frac{p}{2},j+\frac{q}{2}}^{\ell} \cos \theta_{j+\frac{q}{2}} - v_{k+\frac{p}{2},j-\frac{q}{2}}^{\ell} \cos \theta_{j-\frac{q}{2}} + v_{k-\frac{p}{2},j+\frac{q}{2}}^{\ell} \cos \theta_{j+\frac{q}{2}} - v_{k-\frac{p}{2},j-\frac{q}{2}}^{\ell} \cos \theta_{j-\frac{q}{2}}) \right]^{\frac{1}{q}} \right\}
\end{aligned} \tag{11}$$

where σ is given by (8).

4. INPUT

The input file contains four lines. The first input line contains 2 integers:

nx = number of longitudinal points

ny = number of latitudinal points.

The second one contains 3 integers:

dt = time step in seconds

$time_{final}$ = final time in hours

$iplot$ = number of iterations per plot

The third input line contains 2 integers and a real number:

p = stencil in longitudinal direction

q = stencil in latitudinal direction

alf =Pade-type differencing weighting factor

The last input line contains 2 logical variables:

$pstag$ = staggering in p if .true.

$qstag$ = staggering in q if .true.

For example:

```
64 32
100      24  100000
1 1 0.0
.false. .false.
```

5. CODE

```
-----*  
*These lines of code contain the parameter statements for the global  
*definitions of many important parameters.  
-----*  
implicit real*8(a-h,o-z)  
parameter ( imax=128, jmax=64 )  
parameter ( mx=imax*jmax, mxpoi=mx, mxele=mx, mxbou=mx/5, nd=4 )  
parameter ( tol=1.0e-6, g=10.0, rk=0.1 )  
-----*  
*-----*  
*-----*  
*This program solves the Shallow Water Equations  
*on a sphere with Periodic B.C.'s in the latitudinal direction (theta)  
*and longitudinal direction (lambda) using a  
*Staggered Turkel-Zwas Scheme as suggested by B. Neta.  
*Derivatives are obtained via 2nd order differencing with some matching  
*conditions developed by F.X. Giraldo to satisfy continuous derivatives  
*across the poles.  
*Written by F.X. Giraldo on 10/95  
* NRC Fellow  
* Department of Mathematics  
* Naval Postgraduate School  
* Monterey, CA 93940  
-----*  
-----*  
program nturkel  
include 'param.h'  
!global matrices  
real taray(2)
```

```

dimension f(mxpoi)
dimension coord(mxpoi,2)
integer node(imax,jmax), p, q
logical pstag, qstag

        !***primitive variables arrays***
        !u velocity arrays
dimension um(mxpoi),      u0(mxpoi),      up(mxpoi), ui(mxpoi)
        !v velocity arrays
dimension vm(mxpoi),      v0(mxpoi),      vp(mxpoi), vi(mxpoi)
        !phi arrays
dimension phim(mxpoi),    phi0(mxpoi),    phip(mxpoi), phii(mxpoi)

        !Read the Input Variables and create the Grid
call init(phi0,u0,v0,phii,ui,vi,node,coord,f,
$      npoin,xmin,xmax,ymin,ymax,comega,nx,ny,dx,dy,dt,
$      ntime,rade,iplot,omega,alpha,velmax,cfl,p,q,alf,
$      pstag,qstag)

        !Calculate Total Available Potential Energy
call energy(ae,ae0,phi0,u0,v0,npoin,time)
write(*,'("      Energy = ",e12.4)')ae
ae0=ae

time=0.0
pi=4.0*atan(1.0)
open(1,file='phi.out')
open(2,file='u.out')
open(3,file='v.out')
if (mod(ntime,iplot).eq.0) then
  isets=ntime/iplot + 1
else
  isets=ntime/iplot + 2
endif
write(1,*)isets

```

```

write(2,*)isets
write(3,*)isets
call output(phi0,u0,v0,npoin,time,nx,ny,phi_mean)
      !***TIME MARCH
time1=dtimes(taray)
      !Do itime=1 Eulerian steps
do itime=1,ntime
  time=time + dt
  ttime=time/(3600.0)
  write(*,'(" timestep time (hours) = ",i5,2x,e12.4)')itime,ttime
  if (itime.eq.1) then
    call matsuno(phim,phi0,phip,um,u0,up,vm,v0,vp,
$                  coord,f,npoin,dt,dx,dy,node,nx,ny,rade,comega,
$                  alpha,p,q,alf)
  else
    call tzstag(phim,phi0,phip,um,u0,up,vm,v0,vp,coord,
$                  f,npoin,dt,dx,dy,node,nx,ny,rade,comega,
$                  alpha,p,q,alf,pstag,qstag)
  endif
  call sfilter(phip,up,vp,node,nx,ny,dx,dy)
  call time_filter(phim,phi0,phip,um,u0,up,vm,v0,vp,npoin,
$                  itime)
  call update(phim,phi0,phip,um,u0,up,vm,v0,vp,npoin)
  if (mod(itime,iplot).eq.0)
$    call output(phi0,u0,v0,npoin,time,nx,ny,phi_mean)
  call energy(ae,ae0,phi0,u0,v0,npoin,time)
  write(*,'("      Energy = ",e12.4)')ae
end do

time2=etime(taray)
tclock=(taray(1)+taray(2))
write(*,'(" Total CPU time in seconds = ",e12.4)')tclock

```

```

        !Check time for printing output
        if (mod(ntime,iplot).ne.0)
$    call output(phi0,u0,v0,npoin,time,nx,ny,phi_mean)
        close(1)

        !Compute the L2 Error Norm
        call norm(phi0,u0,v0,phii,ui,vi,node,coord,dx,dy,nx,ny,
$              phi_norm,u_norm)
        print*, ' L2 NORM = ',phi_norm,u_norm
        print*, ' dt dx dy velmax = ',dt,dx,dy,velmax
        print*, ' ** CFL = ',cfl
        stop
        end

*-----
*This subroutine calculates the Available Energy of the 2D Shallow Water
*Equations in spherical coordinates
*Written by F.X. Giraldo on 10/95
*-----
subroutine energy(ae,ae0,phi,u,v,npoin,time)
include 'param.h'
        !global arrays
dimension phi(mxpoi), u(mxpoi), v(mxpoi)

ae=0.0
        !loop thru the elements
do ip=1,npoin
        vel2=u(ip)**2 + v(ip)**2
        ae=ae + (phi(ip))*vel2 + phi(ip)**2
end do
ae=ae/(2.0*g)
if (time.gt.0.0) then
        if (ae.gt.1.1*ae0.or.ae.lt.0.9*ae0) then

```

```

        write(*,("      *Fatal Error* 10% Init Energy Exceeded!"))
        write(*,("      Current_Energy Initial_Energy= "
$           ,2(1x,e12.4)))ae,ae0
    endif
    endif
    return
end

*-----*
*This subroutine reads in the input file.
*The info read is: the number of grid points in x and y (nx,ny),
*                  time step, final time, and time steps per plotting,
*                  p, q, alpha
*                  pstag, qstag.
*where .true. means that it is staggered and .false. means it is unstaggered.
*Written by F.X. Giraldo on 10/95
*-----*
subroutine init(phi0,u0,v0,phii,ui,vi,node,coord,f,
$                  npoin,xmin,xmax,ymin,ymax,comega,nx,ny,dx,dy,dt,
$                  ntime,rade,iplot,omega,alpha,velmax,cfl,p,q,alf,
$                  pstag,qstag)
include 'param.h'
dimension coord(mxpoi,2)
dimension phi0(mxpoi), u0(mxpoi), v0(mxpoi)
dimension phii(mxpoi), ui(mxpoi), vi(mxpoi), f(mxpoi)
integer node(imax,jmax), p, q
logical pstag,qstag
        !Read Input File
read(*,*)nx,ny
read(*,*)dt,time_final,iplot
read(*,*)p,q,alf
read(*,*)pstag,qstag
        !check bounds

```

```

if (nx.gt.imax.or.ny.gt.jmax) then
  write(*,(' Error! - Need to Enlarge IMAX and JMAX'))
  write(*,(' nx ny imax jmax = ",4(i3,1x))')nx,ny,imax,jmax
  stop
endif

!Set some constants

pi=4.0*atan(1.0)
rade=6.37e+06
time_final=time_final*3600.0
ntime=nint(time_final/dt)
xmin=0.0
xmax=2.0*pi
ymin=-pi/2.0
ymax=pi/2.0
x1=xmax-xmin
y1=ymax-ymin
dx=x1/(nx)
dy=y1/(ny)
phi_mean=5.768e4
omega=20.0
comega=7.292e-05
velmax=-1e5
alpha_fcor=0.0
alpha=0.0

!set the Initial Conditions

ip=0
do j=1,ny
  olat=ymin + real(j-0.5)*dy
  do i=1,nx
    olon=xmin + real(i-0.5)*dx
    ip=ip+1
    node(i,j)=ip
  enddo
enddo

```

```

        coord(ip,1)=olon
        coord(ip,2)=olat
        f(ip)=2.0*comega*(-cos(olon)*cos(olat)*sin(alpha_fcor) +
$                                sin(olat)*cos(alpha_fcor) )
        u0(ip)=omega*sin(olon)*(sin(olat)**3 -
$                                3*sin(olat)*cos(olat)**2)
        v0(ip)=omega*sin(olat)**2*cos(olon)
        phi0(ip)=phi_mean +
$                                2*comega*rade*omega*sin(olat)**3*cos(olat)*sin(olon)
        phii(ip)=phi0(ip)
        ui(ip)=u0(ip)
        vi(ip)=v0(ip)
        vel1=abs(u0(ip)) + abs(v0(ip)) + sqrt(2*phi0(ip))
        velmax=max(velmax,vel1)
        end do
    end do
    dl=sqrt(dx**2 + dy**2)
    cfl=dt*velmax/(dl*rade)
    print*, ' dt dx dy velmax = ',dt,dx,dy,velmax
    print*, ' ** CFL = ',cfl
    npoin=nx*ny

    return
end

*-----*
*This subroutine solves the 2D Shallow Water Equations in Spherical
*Coordinates using a Staggered Turkel-Zwas Scheme.
*Written by F.X. Giraldo on 10/95
*-----*
        subroutine matsuno(phim,phi0,phip,um,u0,up,vm,v0,vp,
$                                coord,f,npoin,dt,dx,dy,node,nx,ny,rade,comega,
$                                alpha,p,q,alf)

```

```

include 'param.h'

dimension phim(mxpoi), phi0(mxpoi), phip(mxpoi)
dimension um(mxpoi), u0(mxpoi), up(mxpoi)
dimension vm(mxpoi), v0(mxpoi), vp(mxpoi)
dimension coord(mxpoi,2), f(mxpoi)
integer node(imax,jmax), p, q, ph, qh

!Loop through the points and integrate using Forward Time
!and Centered Space...

!Predictor Stage (forward Euler)

ph=p
qh=q
nxh=nx/2

do i=1,nx          !Loop through Longitudinal Nodes
  i1=i-1
  i2=i+1
  i3=i-p
  i4=i+p
  i3h=i-ph
  i4h=i+ph

  !Longitudinal Periodicity
  if (i1.lt.1) i1=i1 + nx
  if (i2.gt.nx) i2=i2 - nx

  !Longitudinal Periodicity -P's and +P's
  if (i3.lt.1) i3=i3 + nx
  if (i4.gt.nx) i4=i4 - nx

  !Longitudinal Periodicity -P/2's and +P/2's
  if (i3h.lt.1) i3h=i3h + nx
  if (i4h.gt.nx) i4h=i4h - nx

  !Loop through Latitudinal Nodes
  do j=1,ny
    j1=j-1
    j2=j+1

```

```

j3=j-q
j4=j+q
j3h=j-qh
j4h=j+qh
j1sign=1
j2sign=1
j3sign=1
j4sign=1
j3hsign=1
j4hsign=1

!South Pole Periodicity

ij1=i
if (j1.lt.1) then
    j1=1
    j1sign=-1
    ij1=ij1 + nxh
    if (ij1.gt.nx) ij1=ij1 - nx
endif

!North Pole Periodicity

ij2=i
if (j2.gt.ny) then
    j2=ny
    j2sign=-1
    ij2=ij2 + nxh
    if (ij2.gt.nx) ij2=ij2 - nx
endif

!South Pole Periodicity -Q's

ij3=i
ippj3=i4
impj3=i3
if (j3.lt.1) then
    j3=1 - j + q

```

```

j3sign=-1
ij3=ij3 + nxh
ippj3=ippj3 + nxh
impj3=impj3 + nxh
if (ij3.gt.nx) ij3=ij3 - nx
if (ippj3.gt.nx) ippj3=ippj3 - nx
if (impj3.gt.nx) impj3=impj3 - nx
endif

!North Pole Periodicity +Q's

ij4=i
ippj4=i4
impj4=i3
if (j4.gt.ny) then
j4=2*ny + 1 - j - q
j4sign=-1
ij4=ij4 + nxh
ippj4=ippj4 + nxh
impj4=impj4 + nxh
if (ij4.gt.nx) ij4=ij4 - nx
if (ippj4.gt.nx) ippj4=ippj4 - nx
if (impj4.gt.nx) impj4=impj4 - nx
endif

!South Pole Periodicity -Q/2's

ij3h=i
ippj3h=i4h
impj3h=i3h
if (j3h.lt.1) then
j3h=1 - j + qh
j3hsign=-1
ij3h=ij3h + nxh
ippj3h=ippj3h + nxh
impj3h=impj3h + nxh

```

```

    if (ij3h.gt.nx) ij3h=ij3h - nx
    if (ippj3h.gt.nx) ippj3h=ippj3h - nx
    if (impj3h.gt.nx) impj3h=impj3h - nx
  endif
      !North Pole Periodicity +Q/2's
  ij4h=i
  ippj4h=ij4h
  impj4h=i3h
  if (j4h.gt.ny) then
    j4h=2*ny + 1 - j - qh
    j4hsign=-1
    ij4h=ij4h + nxh
    ippj4h=ippj4h + nxh
    impj4h=impj4h + nxh
    if (ij4h.gt.nx) ij4h=ij4h - nx
    if (ippj4h.gt.nx) ippj4h=ippj4h - nx
    if (impj4h.gt.nx) impj4h=impj4h - nx
  endif
      !Set up the Node Pointers
      !Centered Diff Grid Points
  ip=node(i,j)
  ip1=node(i1,j)
  ip2=node(i2,j)
  jp1=node(ij1,j1)
  jp2=node(ij2,j2)
      !Turkel-Zwas Grid Points
  ip3=node(i3,j)
  ip4=node(i4,j)
  jp3=node(ij3,j3)
  jp4=node(ij4,j4)
  ip3jp3=node(impj3,j3)
  ip4jp3=node(ippj3,j3)

```

```

ip3jp4=node(impj4,j4)
ip4jp4=node(ippj4,j4)
!Staggered Grid Points
ip3h=node(i3h,j)
ip4h=node(i4h,j)
jp3h=node(ij3h,j3h)
jp4h=node(ij4h,j4h)
ip3hjp3h=node(impj3h,j3h)
ip4hjp3h=node(ippj3h,j3h)
ip3hjp4h=node(impj4h,j4h)
ip4hjp4h=node(ippj4h,j4h)
!Longitudes and Latitudes
olon=coord(ip,1)
olat=coord(ip,2)
olonpp=olon + p*dx
olonmp=olon - p*dx
olonpq=olon
if (j4sign.eq.-1) olonpq=olonpq + pi
olatpq=olat + q*dy
olonmq=olon
if (j3sign.eq.-1) olonmq=olonmq + pi
olatmq=olat - q*dy
!Staggered Longitudes and Latitudes
olonpqh=olon
if (j4hsign.eq.-1) olonpqh=olonpqh + pi
olatpqh=olat + qh*dy
olonmqh=olon
if (j3hsign.eq.-1) olonmqh=olonmqh + pi
olatmqh=olat - qh*dy
!Coriolis Force
fip=2*comega*(-cos(olon)*cos(olat)*sin(alpha) +
$                                sin(olat)*cos(alpha) )

```

```

        fip4=2*comega*( -cos(olonpp)*cos(olat)*sin(alpha) +
$                               sin(olat)*cos(alpha) )
        fip3=2*comega*( -cos(olonmp)*cos(olat)*sin(alpha) +
$                               sin(olat)*cos(alpha) )
        fjp4=2*comega*( -cos(olonpq)*cos(olatpq)*sin(alpha) +
$                               sin(olatpq)*cos(alpha) )
        fjp3=2*comega*( -cos(olonmq)*cos(olatmq)*sin(alpha) +
$                               sin(olatmq)*cos(alpha) )

        fip=f(ip)
        fip4=f(ip4)
        fip3=f(ip3)
        fjp4=f(jp4)
        fjp3=f(jp3)

        !integrate PHI
        phip(ip)=phi0(ip)
$ - dt*u0(ip)/(rade*cos(olat))*(phi0(ip2)-phi0(ip1))/(2*dx)
$ - dt*v0(ip)/(rade)*(phi0(jp2)-phi0(jp1))/(2*dy)
$ - dt*phi0(ip)/(rade*cos(olat))*(1.0-alf)*(u0(ip4h)-u0(ip3h))/(2*ph*dx) +
$           (j4hsign*v0(jp4h)*cos(olatpqh) -
$           j3hsign*v0(jp3h)*cos(olatmqh))/(2*qh*dy) +
$alf/2*( (j4hsign*u0(ip4hjp4h)-j4hsign*u0(ip3hjp4h))/(2*ph*dx) +
$           (j3hsign*u0(ip4hjp3h)-j3hsign*u0(ip3hjp3h))/(2*ph*dx) +
$           (j4hsign*v0(ip4hjp4h)*cos(olatpqh) -
$           j3hsign*v0(ip4hjp3h)*cos(olatmqh))/(2*qh*dy) +
$           (j4hsign*v0(ip3hjp4h)*cos(olatpqh) -
$           j3hsign*v0(ip3hjp3h)*cos(olatmqh))/(2*qh*dy) ) )

c
        phip(ip)=phi0(ip)
        !integrate U
        up(ip)=u0(ip)
$ - dt*u0(ip)/(rade*cos(olat))*(u0(ip2)-u0(ip1))/(2*dx)

```

```

$      - dt*v0(ip)/rade*(j2sign*u0(jp2)-j1sign*u0(jp1))/(2*dy)
$      - dt/(rade*cos(olat))*(phi0(ip4h)-phi0(ip3h))/(2*ph*dx)
$      + dt*(
$          (1.0-alf)*(fip + u0(ip)/rade*tan(olat))*v0(ip) +
$          alf/2*(fip4 + u0(ip4)/rade*tan(olat))*v0(ip4) +
$          alf/2*(fip3 + u0(ip3)/rade*tan(olat))*v0(ip3) )

c      up(ip)=u0(ip)
           !integrate V
vp(ip)=v0(ip)
$      - dt*u0(ip)/(rade*cos(olat))*(v0(ip2)-v0(ip1))/(2*dx)
$      - dt*v0(ip)/rade*(j2sign*v0(jp2)-j1sign*v0(jp1))/(2*dy)
$      - dt/rade*( phi0(ip4h)-phi0(ip3h) )/(2*qh*dy)
$      - dt*(
$          (1.0-alf)*(fip + u0(ip)/rade*tan(olat))*u0(ip) +
$          alf/2*(fjp4 +
$                  j4sign*u0(ip4)/rade*tan(olatpq))*j4sign*u0(ip4) +
$          alf/2*(fjp3 +
$                  j3sign*u0(ip3)/rade*tan(olatmq))*j3sign*u0(ip3) )

c      vp(ip)=v0(ip)
end do
end do
return
end

*-----*
*This subroutine computes the L2 Norm
*for the Geopotential and Velocity using
*a Trapezoid Rule Integration.
*Written by F.X. Giraldo on 10/95
*-----*
subroutine norm(phi0,u0,v0,phii,ui,vi,node,coord,dx,dy,nx,ny,

```

```

$           phi_norm,u_norm)

include 'param.h'

dimension phi0(mxpoi), u0(mxpoi), v0(mxpoi)
dimension phii(mxpoi), ui(mxpoi), vi(mxpoi)
dimension coord(mxpoi,2), phih(128,64), uh(128,64), vh(128,64)
integer node(imax,jmax)

pi=4.0*atan(1.0)
open(10,file='phih.out')
open(20,file='uh.out')
open(30,file='vh.out')
do j=1,64
  do i=1,128
    read(10,*)phih(i,j)
    read(20,*)uh(i,j)
    read(30,*)vh(i,j)
  end do
end do
close(10)
close(20)
close(30)
do j=1,ny
  do i=1,nx
    ip=node(i,j)
    ui(ip)=uh(2*i-1,2*j-1)
    vi(ip)=vh(2*i-1,2*j-1)
    phii(ip)=phih(2*i-1,2*j-1)
  end do
end do

phi_top=0.0
phi_bot=0.0

```

```

u_top=0.0
u_bot=0.0
do j=1,ny-1
  do i=1,nx-1
    i1=node(i,j)
    i2=node(i+1,j)
    i3=node(i+1,j+1)
    i4=node(i,j+1)
    olat1=coord(i1,2)
    olat2=coord(i2,2)
    olat3=coord(i3,2)
    olat4=coord(i4,2)
    phi1=(phi0(i1) - phii(i1))*cos(olat1)
    u1=(u0(i1) - ui(i1))*cos(olat1)
    v1=(v0(i1) - vi(i1))*cos(olat1)
    phi2=(phi0(i2) - phii(i2))*cos(olat2)
    u2=(u0(i2) - ui(i2))*cos(olat2)
    v2=(v0(i2) - vi(i2))*cos(olat2)
    phi3=(phi0(i3) - phii(i3))*cos(olat3)
    u3=(u0(i3) - ui(i3))*cos(olat3)
    v3=(v0(i3) - vi(i3))*cos(olat3)
    phi4=(phi0(i4) - phii(i4))*cos(olat4)
    u4=(u0(i4) - ui(i4))*cos(olat4)
    v4=(v0(i4) - vi(i4))*cos(olat4)
    phi=dx*dy*(phi1 + phi2 + phi3 + phi4)/4
    phie=dx*dy*(phii(i1) + phii(i2) + phii(i3) + phii(i4))/4
    u=dx*dy*(u1 + u2 + u3 + u4)/4
    ue=dx*dy*(ui(i1) + ui(i2) + ui(i3) + ui(i4))/4
    v=dx*dy*(v1 + v2 + v3 + v4)/4
    ve=dx*dy*(vi(i1) + vi(i2) + vi(i3) + vi(i4))/4

    phi_top=phi_top + ( phi )**2

```

```

phi_bot=phi_bot + ( phie )**2
u_top=u_top + ( u )**2 + ( v )**2
u_bot=u_bot + ( ue )**2 + ( ve )**2
end do
end do
phi_norm=1.0/(4.0*pi)*sqrt(phi_top/phi_bot)
u_norm=1.0/(4.0*pi)*sqrt(u_top/u_bot)

return
end

*-----*
*This subroutine writes the output. It is currently set only to
*print the geopotential and wind velocities at each node point.
*Written by F.X. Giraldo on 10/95
*-----*

subroutine output(phi,u,v,npoin,time,nx,ny,phi_mean)
include 'param.h'
dimension phi(mxpoi), u(mxpoi), v(mxpoi)

pi=4.0*atan(1.0)
dtime=time/3600.0
write(1,'(2(i6,1x),e16.8)')nx,ny,dtime
write(1,'(e12.4)')(phi(ip), ip=1,npoin)
write(2,'(2(i6,1x),e16.8)')nx,ny,dtime
write(2,'(e12.4)')(u(ip), ip=1,npoin)
write(3,'(2(i6,1x),e16.8)')nx,ny,dtime
write(3,'(e12.4)')(v(ip), ip=1,npoin)
return
end

*-----*
*This subroutine performs the Robert time filtering using a
*Laplacian type time-diffusion term that smoothens the values spatially.

```

*Written by F.X. Giraldo on 10/95

-----*

```
subroutine sfilter(phi0,u0,v0,node,nx,ny,dx,dy)
  include 'param.h'
  dimension phi0(mxpoi), phip(mxpoi)
  dimension u0(mxpoi), up(mxpoi)
  dimension v0(mxpoi), vp(mxpoi)
  integer node(imax,jmax)

  do i=1,nx
    i1=i-1
    i2=i+1
    if (i1.lt.1) i1=nx
    if (i2.gt.nx) i2=1
    do j=1,ny
      if (j.gt.2.or.j.lt.ny-1) goto 100
      j1=j-1
      j2=j+1

      !Set up the Node Pointers
      ip=node(i,j)
      ip1=node(i1,j)
      ip2=node(i2,j)
      jp1=node(i,j1)
      jp2=node(i,j2)

      phi0_xx=( phi0(ip2) - 2*phi0(ip) + phi0(ip1) )/(dx*dx)
      u0_xx=( u0(ip2) - 2*u0(ip) + u0(ip1) )/(dx*dx)
      v0_xx=( v0(ip2) - 2*v0(ip) + v0(ip1) )/(dx*dx)
      phi0_yy=( phi0(jp2) - 2*phi0(ip) + phi0(jp1) )/(dy*dy)
      u0_yy=( u0(jp2) - 2*u0(ip) + u0(jp1) )/(dy*dy)
      v0_yy=( v0(jp2) - 2*v0(ip) + v0(jp1) )/(dy*dy)
```

```

        !South Pole Periodicity
if (j1.lt.1) then
    j1=1
    ij1=i + nx/2
    if (ij1.gt.nx) ij1=ij1 - nx
    jp1=node(ij1,j1)
    phi0_yy=( phi0(jp2) - 2*phi0(ip) + phi0(jp1) )/(dy*dy)
    u0_yy=( u0(jp2) - 2*u0(ip) + u0(jp1) )/(dy*dy)
    v0_yy=( v0(jp2) - 2*v0(ip) + v0(jp1) )/(dy*dy)
endif

        !North Pole Periodicity
if (j2.gt.ny) then
    j2=ny
    ij2=i + nx/2
    if (ij2.gt.nx) ij2=ij2 - nx
    jp2=node(ij2,j2)
    phi0_yy=( phi0(jp2) - 2*phi0(ip) + phi0(jp1) )/(dy*dy)
    u0_yy=( u0(jp2) - 2*u0(ip) + u0(jp1) )/(dy*dy)
    v0_yy=( v0(jp2) - 2*v0(ip) + v0(jp1) )/(dy*dy)
endif

    phip(ip)=phi0(ip) + rk*( phi0_xx + phi0_yy )
    up(ip)=u0(ip) + rk*( u0_xx + u0_yy )
    vp(ip)=v0(ip) + rk*( v0_xx + v0_yy )
100    continue
end do
end do

do i=1,nx
    do j=1,ny
        if (j.gt.2.or.j.lt.ny-1) goto 200

```

```

        ip=node(i,j)
        phi0(ip)=phip(ip)
        u0(ip)=up(ip)
        v0(ip)=vp(ip)

200      continue
        end do
        end do

        return
        end

*-----*
*This subroutine performs the Robert time filtering using a
*Laplacian type time-diffusion term that smoothens the values temporally.
*Written by F.X. Giraldo on 10/95
*-----*

subroutine time_filter(phim,phi0,phip,um,u0,up,vm,v0,vp,npoin,
$                      itime)
  include 'param.h'
  dimension phim(mxpoi), phi0(mxpoi), phip(mxpoi)
  dimension um(mxpoi), u0(mxpoi), up(mxpoi)
  dimension vm(mxpoi), v0(mxpoi), vp(mxpoi)

  if (itime.eq.2) then
    do ip=1,npoin
      phi0(ip)=phi0(ip) + rk*( phip(ip) - phi0(ip) )
      u0(ip)=u0(ip) + rk*( up(ip) - u0(ip) )
      v0(ip)=v0(ip) + rk*( vp(ip) - v0(ip) )
    end do
  else if (itime.gt.2) then
    do ip=1,npoin
      phi0(ip)=phi0(ip) + rk*( phip(ip) - 2*phi0(ip) + phim(ip) )
      u0(ip)=u0(ip) + rk*( up(ip) - 2*u0(ip) + um(ip) )
    end do
  end if
end

```

```

    v0(ip)=v0(ip) + rk*( vp(ip) - 2*v0(ip) + vm(ip) )
  end do
  endif
  return
end

*-----*
*This subroutine solves the 2D Shallow Water Equations in Spherical
*Coordinates using a Staggered Turkel-Zwas Scheme.
*Written by F.X. Giraldo on 10/95
*-----*

subroutine tzstag(phim,phi0,phip,um,u0,up,vm,v0,vp,coord,
$                   f,npoin,dt,dx,dy,node,nx,ny,rade,comega,
$                   alpha,p,q,alf,pstag,qstag)
include 'param.h'
dimension phim(mxpoi), phi0(mxpoi), phip(mxpoi)
dimension um(mxpoi), u0(mxpoi), up(mxpoi)
dimension vm(mxpoi), v0(mxpoi), vp(mxpoi)
dimension coord(mxpoi,2), f(mxpoi)
integer node(imax,jmax), p, q, ph, qh
logical pstag, qstag
!Loop through the points and integrate using Forward Time
!and Centered Space...
!Predictor Stage (forward Euler)
if (pstag) then
  ph=p/2
else
  ph=p
endif
if (qstag) then
  qh=q/2
else
  qh=q

```

```

        endif

        alfh=0
        alfu=0
        alfv=0
        nxh=nx/2

        do i=1,nx      !Loop through Longitudinal Nodes
            i1=i-1
            i2=i+1
            i3=i-p
            i4=i+p
            i3h=i-ph
            i4h=i+ph
                !Longitudinal Periodicity
            if (i1.lt.1) i1=i1 + nx
            if (i2.gt.nx) i2=i2 - nx
                !Longitudinal Periodicity -P's and +P's
            if (i3.lt.1) i3=i3 + nx
            if (i4.gt.nx) i4=i4 - nx
                !Longitudinal Periodicity -P/2's and +P/2's
            if (i3h.lt.1) i3h=i3h + nx
            if (i4h.gt.nx) i4h=i4h - nx
                !Loop through Latitudinal Nodes
        do j=1,ny
            j1=j-1
            j2=j+1
            j3=j-q
            j4=j+q
            j3h=j-qh
            j4h=j+qh
            j1sign=1
            j2sign=1
            j3sign=1

```

```

j4sign=1
j3hsign=1
j4hsign=1
!South Pole Periodicity
ij1=i
if (j1.lt.1) then
    j1=1
    j1sign=-1
    ij1=ij1 + nxh
    if (ij1.gt.nx) ij1=ij1 - nx
endif
!North Pole Periodicity
ij2=i
if (j2.gt.ny) then
    j2=ny
    j2sign=-1
    ij2=ij2 + nxh
    if (ij2.gt.nx) ij2=ij2 - nx
endif
!South Pole Periodicity -Q's
ij3=i
ippj3=i4
impj3=i3
if (j3.lt.1) then
    j3=1 - j + q
    j3sign=-1
    ij3=ij3 + nxh
    ippj3=ippj3 + nxh
    impj3=impj3 + nxh
    if (ij3.gt.nx) ij3=ij3 - nx
    if (ippj3.gt.nx) ippj3=ippj3 - nx
    if (impj3.gt.nx) impj3=impj3 - nx

```

```

        endif

        !North Pole Periodicity +Q's

        ij4=i
        ippj4=i4
        impj4=i3
        if (j4.gt.ny) then
            j4=2*ny + 1 - j - q
            j4sign=-1
            ij4=ij4 + nxh
            ippj4=ippj4 + nxh
            impj4=impj4 + nxh
            if (ij4.gt.nx) ij4=ij4 - nx
            if (ippj4.gt.nx) ippj4=ippj4 - nx
            if (impj4.gt.nx) impj4=impj4 - nx
        endif

        !South Pole Periodicity -Q/2's

        ij3h=i
        ippj3h=i4h
        impj3h=i3h
        if (j3h.lt.1) then
            j3h=1 - j + qh
            j3hsign=-1
            ij3h=ij3h + nxh
            ippj3h=ippj3h + nxh
            impj3h=impj3h + nxh
            if (ij3h.gt.nx) ij3h=ij3h - nx
            if (ippj3h.gt.nx) ippj3h=ippj3h - nx
            if (impj3h.gt.nx) impj3h=impj3h - nx
        endif

        !North Pole Periodicity +Q/2's

        ij4h=i
        ippj4h=i4h

```

```

impj4h=i3h
if (j4h.gt.ny) then
  j4h=2*ny + 1 - j - qh
  j4hsign=-1
  ij4h=ij4h + nxh
  ippj4h=ippj4h + nxh
  impj4h=impj4h + nxh
  if (ij4h.gt.nx) ij4h=ij4h - nx
  if (ippj4h.gt.nx) ippj4h=ippj4h - nx
  if (impj4h.gt.nx) impj4h=impj4h - nx
endif

!Set up the Node Pointers
!Centered Diff Grid Points
ip=node(i,j)
ip1=node(i1,j)
ip2=node(i2,j)
jp1=node(ij1,j1)
jp2=node(ij2,j2)

!Turkel-Zwas Grid Points
ip3=node(i3,j)
ip4=node(i4,j)
jp3=node(ij3,j3)
jp4=node(ij4,j4)
ip3jp3=node(impj3,j3)
ip4jp3=node(ippj3,j3)
ip3jp4=node(impj4,j4)
ip4jp4=node(ippj4,j4)

!Staggered Grid Points
ip3h=node(i3h,j)
ip4h=node(i4h,j)
jp3h=node(ij3h,j3h)
jp4h=node(ij4h,j4h)

```

```

ip3hjp3h=node(impj3h,j3h)
ip4hjp3h=node(ippj3h,j3h)
ip3hjp4h=node(impj4h,j4h)
ip4hjp4h=node(ippj4h,j4h)

!Longitudes and Latitudes
olon=coord(ip,1)
olat=coord(ip,2)
olatpq=olat + q*dy
olatmq=olat - q*dy

!Staggered Longitudes and Latitudes
olatpqh=olat + qh*dy
olatmqh=olat - qh*dy

!Coriolis Force
fip=f(ip)
fip4=f(ip4)
fip3=f(ip3)
fjp4=f(jp4)
fjp3=f(jp3)

!integrate PHI
phip(ip)=phim(ip)
$ - dt*u0(ip)/(rade*cos.olat)*(phi0(ip2)-phi0(ip1))/dx
$ - dt*v0(ip)/(rade)*(phi0(jp2)-phi0(jp1))/dy
$ - dt*phi0(ip)/(rade*cos.olat)*(
$ (1.0-alf)*((u0(ip4h)-u0(ip3h))/(ph*dx) +
$ (j4hsign*v0(jp4h)*cos.olatpqh) -
$ j3hsign*v0(jp3h)*cos.olatmqh)/(qh*dy) ) +
$ alf/2*((j4hsign*u0(ip4hjp4h)-j4hsign*u0(ip3hjp4h))/(ph*dx) +
$ (j3hsign*u0(ip4hjp3h)-j3hsign*u0(ip3hjp3h))/(ph*dx) +
$ (j4hsign*v0(ip4hjp4h)*cos.olatpqh) -
$ j3hsign*v0(ip4hjp3h)*cos.olatmqh)/(qh*dy) +
$ (j4hsign*v0(ip3hjp4h)*cos.olatpqh) -
$ j3hsign*v0(ip3hjp3h)*cos.olatmqh)/(qh*dy) ) )

```

```

c      phip(ip)=phi0(ip)
          !integrate U
up(ip)=um(ip)
$      - dt*u0(ip)/(rade*cos(olat))*(u0(ip2)-u0(ip1))/dx
$      - dt*v0(ip)/rade*(j2sign*u0(jp2)-j1sign*u0(jp1))/dy
$      - dt/(rade*cos(olat))*(phi0(ip4h)-phi0(ip3h))/(ph*dx)
$      + 2*dt*(
$          (1.0-alf)*(fip + u0(ip)/rade*tan(olat))*v0(ip) +
$          alf/2*(fip4 + u0(ip4)/rade*tan(olat))*v0(ip4) +
$          alf/2*(fip3 + u0(ip3)/rade*tan(olat))*v0(ip3) )
c      up(ip)=u0(ip)
          !integrate V
vp(ip)=vm(ip)
$      - dt*u0(ip)/(rade*cos(olat))*(v0(ip2)-v0(ip1))/dx
$      - dt*v0(ip)/rade*(j2sign*v0(jp2)-j1sign*v0(jp1))/dy
$      - dt/rade*( phi0(jp4h)-phi0(jp3h) )/(qh*dy)
$      - 2*dt*(
$          (1.0-alfv)*(fip
$              + u0(ip)/rade*tan(olat))*u0(ip)
$          + alfv/2*(fjp4
$              + j4sign*u0(jp4)/rade*tan(olatpq))*j4sign*u0(jp4)
$          + alfv/2*(fjp3
$              + j3sign*u0(jp3)/rade*tan(olatmq))*j3sign*u0(jp3) )
c      vp(ip)=v0(ip)
          end do
      end do
      return
  end
*-----*
*This subroutine updates the arrays PHIM,UM,VM,PHIO,U0,V0,
*Written by F.X. Giraldo on 10/95
*-----*

```

```

subroutine update(phim,phi0,phip,um,u0,up,vm,v0,vp,npoin)
include 'param.h'
dimension phim(mxpoi), phi0(mxpoi), phip(mxpoi)
dimension um(mxpoi),   u0(mxpoi),   up(mxpoi)
dimension vm(mxpoi),   v0(mxpoi),   vp(mxpoi)
           !Loop through all the nodes and update
do ip=1,npoin
           !Update F(x-2*alpha,t-dt)=F(x-alpha,t)
phim(ip)=phi0(ip)
um(ip)=u0(ip)
vm(ip)=v0(ip)
           !Update F(x-alpha,t)=F(x,t+dt)
phi0(ip)=phip(ip)
u0(ip)=up(ip)
v0(ip)=vp(ip)
end do

return
end

```

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